ABSTRACTS

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TALKS

Tripartite probabilistic quantum teleportation in the presence of noise

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We investigate probabilistic quantum teleportation through noisy tripartite qubit channels. In particular, we discuss how the efficiency of the quantum teleportation protocol is affected when the qubits involved in the protocol are subjected to selected local noise or decoherence. As a quantum channel several tripartite qubit-states are taken in which a part or all of the qubits are subjected to the same or different types of noise. We study all types of noise usually encountered in real-world implementations of quantum communication protocols, namely, the bit-flip, phase-flip (phase damping), depolarizing and generalized amplitude-damping noise. Furthermore, the above analyses is supported by expansive investigation of various tripartite entanglement measures.

State mapping between two cavity modes with wavelength conversion

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We show that an ensemble of four-level atoms in the diamond configuration makes it possible to perform a state-mapping operation, which transfers an arbitrary state of light from one cavity mode to the second mode of different frequency [1]. The diamond configuration also makes it possible to fully control the state-mapping operation, *i.e.*, we can easily switch the mapping on and off on demand just by switching lasers on and off. The ensemble of four-level atoms in the diamond configuration interacting with two quantized cavity modes and two classical laser fields is a complex system. However, we show that under certain conditions the evolution of this complex system can be described by a simple effective Hamiltonian. Finally, we study the effect of non-zero spontaneous decay rates of excited states on the state-mapping operation.

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Coupling constant triggered spectral transitions

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Even a small change of the coupling constant may have profound influence of the spectrum. A much more subtle question is whether this could happen if the change affects the potential only on a set of a „small" or even zero measure. The aim of this talk is to provide an affirmative answer, demonstrating it on several classes of Schrödinger operators with potentials that are below unbounded but their negative part is localized in narrow channels. A prototype of such a behavior can be found in Smilansky-Solomyak model devised to illustrate that an an irreversible behavior is possible even if the heat bath to which the systems is coupled has a finite number of degrees of freedom. We review its properties and analyze a regular version of this model, as well as another system in which $x^p y^p$ potential is amended by a negative radially symmetric term. All of them have the common property that they exhibit an abrupt parameter-dependent spectral transition: if the coupling constant exceeds a critical value the spectrum changes from a below bounded, partly or fully discrete, to the continuous one covering the whole real axis. We also discuss resonance effects in such models. The results come from a common work with Diana Barseghyan, Andrii Khrabustovskyi, Vladimir Lotoreichik and Miloš Tater.

Analysis of the spectral Variability of Cyg X-1 using INTEGRAL data

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The studies of the spectral states are an important diagnostic tool of the black hole (BH) binary systems and Cyg X-1 is one of the most well analyzed object in this respect. We present a novel approach to the spectral state identification based on hard X-ray emission, which results in much less misclassified observations between the three main spectral states (soft, intermediate and hard) of Cyg X-1. Using all of the 14-years INTEGRAL data publicly available so far, we performed a comprehensive analysis over a broad range of the Cyg X-1 spectral states. The INTEGRAL/IBIS data were split in 14 groups, with almost equal total exposure times. These 14 broad-band INTEGRAL/ISGRI and INGTEGRAL/JEMX spectral datasets were analyzed with the two most advanced Comptonization models implemented in the XSPEC fitting package, COMPPS and EQPAIR with the soft photon temperature left to vary but also fixed to a specific value. The spectral analysis of the high quality substate spectra allowed us to find a correspondence between the parameters of the physical models and the phenomenological classification based on the INTEGRAL/IBIS count rates. To determine a full range of the spectral parameters we analyzed almost 1500 ISGRI + JEMX simultaneous spectra of individual INTEGRAL pointings (0.5 — 2 hours exposure time). Based on the results of this exhaustive spectral analysis, we investigated correlations between various spectral and other physical parameters of Cyg X-1 such as the Compton amplification factor, plasma temperature, soft photons temperature, reflection strength e.t.c. as the spectral shapes evolve from the soft state to the hard state. We also use semi-simultaneous data from INTEGRAL/ISGRI, INTEGRAL/JEMX and INTEGRAL/PICsIT

X-ray and AMI radio observations to search for correlations between them. Also, using the sensitivity of INTEGRAL/ISGRI we apply our classification criteria to the SWIFT/BAT monitor, which provides daily Cyg X-1 data. Thus, a quick and reliable classification of the Cyg X-1 observations can be accomplished.

In summary, we found that the phenomenological classification of the Cyg X-1 spectral states based on the hard X-ray count rates allows for a more decisive identification of the states than the ones based on the soft X-rays. In addition, a thorough spectral modelling shows that the hard X-ray categorization unambiguously corresponds to distinct physical parameters values of the Cyg X-1 spectra determined for different states. Furthermore, we propose to use the SWIFT/BAT monitor for the classification of Cyg X-1, using the INTEGRAL/IBIS hard X-ray range and criteria.

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Parameters of noncommutativity in noncommutative phase space

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We consider a space with noncommutativity of coordinates and noncommutativity of momenta

$$[X_i, X_j] = i\hbar\theta_{ij}, \tag{1}$$

$$[X_i, P_j] = i\hbar\delta_{ij}, \tag{2}$$

$$[P_i, P_j] = i\hbar\eta_{ij}, \tag{3}$$

where θ_{ij} and η_{ij} are parameters of noncommutativity.

We show that coordinates in noncommutative phase space can not be considered as kinematic variables in noncommutative phase space because of their dependence on the mass. Also, noncommutative momenta are not proportional to a mass as it has to be. We have solved the problems with the help of conditions on the parameters of noncommutativity [1]. It is important that on the same conditions the weak equivalence principle is recovered, the properties of kinetic energy are preserved, and the motion of the center-of-mass of composite system and relative motion are independent in noncommutative phase space [2].

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Pyramid-shaped heavy and superheavy nuclei?

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In this talk I will present the results of our search for the effects of tetrahedral symmetry over a range of almost 3000 heavy and superheavy nuclei, Z=82-126, including odd and odd-odd systems. Our analysis are performed within the macroscopic-microscopic model, based on the deformed Woods-Saxon single-particle potential and the Yukawa-plus-exponential macroscopic energy. All parameters of the model, well tested during many years in the region of the heaviest nuclei, are kept unchanged. The shell and pairing correction for an odd-nucleon system is done by blocking the lowest-lying quasiparticle state.
In our calculations we look for the energy minima with a nonzero tetrahedral distortion, both absolute and conditional-with the quadrupole distortion constrained to zero. In order to assure reliability of our results we include the ten most important deformation parameters in the energy minimization procedure.
In case of 40 superheavy nuclei from the range: Z=119-126 and N=173-188, we find visible (150-730 keV) lowering its ground-state energy due to inclusion β_{32} parameter, which in the first approximation can describe tetrahedral shapes. However, this effect is not the result appearing of pure tetrahedral symmetry at the ground state, but results from the combination of nonzero tetrahedral distortion with some others deformation parameters.

Quantum steering and other correlations in a three-qubit systems

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We consider a class of quantum states which describe three-qubit systems. In our studies, we concentrate on the possibility of producing of bipartite steering in three-qubit models. To quantify the steering in two-qubit subsystems, we apply the parameters based on Cavalcanti inequality [1]. We discuss the conditions determining when steering effects can appear and show its relations to the degree of bipartite coherences [2, 3] and the entanglement measured by the concurrence [4, 5]. Additionally, we determine the borders between various classes of quantum states characterized by the parameters describing steering, coherence and bipartite entanglement.

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Spectroscopic properties and quantum efficiency of the Eu-doped borate glasses

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Spectroscopic and luminescent properties of the Eu-doped borate glasses have been investigated using electron paramagnetic resonance (EPR), optical spectroscopy (absorption, photoluminescence, decay kinetics, quantum yield), modified Judd-Ofelt analysis, and X-ray diffraction (XRD) techniques.

Borate glasses with $Li_2B_4O_7$ ($Li_2O - 2B_2O_3$), CaB_4O_7 ($CaO - 2B_2O_3$), and $LiCaBO_3$ ($0.5Li_2O - CaO - 0.5B_2O_3$) compositions containing 0.5 and 1.0 mol. % Eu_2O_3 have been obtained by standard glass synthesis according to technological conditions, described in [1]. The XRD patterns of obtained materials confirm their disordered glassy structure.

Characteristic signals of the Eu^{2+} ($4f^7$, $^8S_{7/2}$) paramagnetic centres were not observed in the EPR spectra of investigated glasses. Thus, europium impurity is incorporated into the $Li_2B_4O_7$, CaB_4O_7, and $LiCaBO_3$ glass networks as Eu^{3+} ($4f^6$, $^7F_0$) ions, exclusively. Optical absorption spectra of the Eu-doped borate glasses consist of several narrow bands in the visible and infrared spectral ranges, which are characteristic for Eu^{3+} ions.

The photoluminescence spectra registered under excitation with λ_{exc} = 393 nm ($^7F_0 \rightarrow {}^5L_6$ transition) are closely similar and consist of emission bands peaked at 577, 591, 611, 651, and 701 nm, which correspond to the $^5D_0 \rightarrow {}^7F_0$, $^7F_1$, $^7F_2$, $^7F_3$, and $^7F_4$ transitions of Eu^{3+} ions. Luminescence kinetics of the Eu^{3+} centres in the investigated glasses is satisfactory described by single exponential decay with lifetimes in the 2.04 – 2.26 ms range.

The Judd-Ofelt intensity parameters (Ω_λ) for Eu^{3+} centres in the investigated glasses were calculated from their luminescence emission spectra. Radiative lifetime and internal quantum efficiency were estimated for observed emission transitions of the Eu^{3+} centres. External quantum yield of luminescence was measured experimentally via an absolute method using an integrating sphere. The calculated internal quantum efficiency ($\sim 50\%$) and measured external quantum yield ($\sim 11\%$) of the Eu^{3+} luminescence

in the investigated borate glasses are comparable with corresponding values, obtained for other Eu^{3+}-doped oxide materials.

Peculiarities of local structure of the Eu^{3+} centres in the network of investigated borate glasses have been discussed based on obtained spectroscopic results and XRD structural data.

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Quantum layers with resonators

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We consider two and three dimensional models of quantum layers with a delta potential preserving symmetry and inducing embedded eigenvalues. We show that after the breaking symmetry the embedded eigenvalues turn to resonances determined by the second sheet poles of the resolvent. We analyze various types of "perturbants" which lead to breaking symmetry. Finally, we discuss how the widths of resonances depend on breaking symmetry parameters.

Thermal absorption in the pulsar radio spectra: the case of the binary system B1259−63/LS 2883

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The number of the so called gigahertz-peaked spectra pulsars (GPS) is constantly growing. So far we know about 25 objects that show high-frequency turnovers (at the frequency around 1 GHz). Interestingly, most of these sources can be found inside some peculiar environments: supernova remnants, pulsar wind nebulae and dense HII regions. We believe that these environments are responsible for the GPS pulsars spectra appearance, and the "primary suspect" is the thermal free-free absorption occurring in the relatively dense matter surrounding the pulsar. The binary system of PSR B1259−63 and Be star LS 2883 provides a window into the GPS phenomenon. The spectrum of B1259−63 at various orbital phases mimics the spectrum of a GPS pulsar. Here we present our detailed model of thermal absorption in this binary system.

Preparation of quantum gates on two spin-1/2 particles

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We consider the quantum evolution of two spins described by an anisotropic Heisenberg interaction in the magnetic field. Using the evolution operator of this system the conditions for realization of some two-qubit quantum gates are obtained. The fidelity for realization of these gates is calculated. Finally, we propose the physical implementation of quantum gates on a system of the atom which has a nuclear spin 1/2 and a valence electron.

Entanglement properties of highly symmetric qudit states

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Quantum entanglement is one of the striking features of quantum theory, which distinguish quantum from the classical world. For that reason, the characterization of entanglement properties in quantum states is an important task from theoretical and practical point of view. While the calculation of entanglement measures for pure two-qudit states is relatively straightforward, it becomes much more complicated in the case of mixed two-qudit states. The basic idea of dealing with this problem is simply to focus on a subgroup of states, which obey some symmetrical conditions. Here we discuss a recently introduced highly symmetric qudit states family with incomplete permutation symmetry[1, 2]. This family contains both pure and mixed states obeying such symmetry and in general is described by five real parameters. For such states we perform an extensive analytical analysis of various conditions of separability and the entanglement classification with respect to stochastic local operations and classical communication. Furthermore our results can by used for any arbitrary quantum state by application of twirling operator.

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Spectroscopy and luminescence of borate glasses, co-doped with Gd and Ag

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Borate glasses of high optical quality with $Li_2B_4O_7$ and $LiKB_4O_7$ basic compositions doped with Gd and Ag as well as co-doped with Gd and Ag have been detailed investigated using electron paramagnetic resonance (EPR) and optical spectroscopy methods. The studied glasses were obtained by standard synthesis of borate glasses, described in [1]. The Gd impurity was introduced in the $Li_2B_4O_7$ and $LiKB_4O_7$ glass compositions as Gd_2O_3 in amount 1.0 mol. %. The Ag impurity was introduced into the network of investigated glasses as $AgNO_3$ salt and as metallic highly-dispersed silver in amount 2.0 mol. %. In all Gd-doped glasses at room temperature (RT) was observed EPR spectrum of the Gd^{3+} ($^8S_{7/2}$, $4f^7$) paramagnetic ions that is typical for all glasses and practically independent of the basic glass composition [2]. In the Ag-doped and Gd-Ag co-doped glasses was observed complex broad EPR signal that corresponds to the Ag^{2+} ($4d^9$), Ag^0 ($5s^1$), and Ag_2^+ centres [3, 4, 5] as well as Ag metallic nanoclusters.

In the Gd-doped $Li_2B_4O_7$ and other glasses at RT besides complex broad emission band of intrinsic luminescence, described in [6], is observed weak narrow UV emission band at 311 nm under excitation with 273 nm and 252 nm. The emission band at 311 nm is extremely efficient at excitation with 273 nm. Emission band peaked at 311 nm was assigned to the $^6P_{7/2} \rightarrow {}^8S_{7/2}$ intraconfiguration $4f-4f$ transition of the Gd^{3+} ions [7]. In the luminescence excitation spectra of the Gd-doped borate glasses are well observed three characteristic group of bands, which correspond to the Gd^{3+} transitions: $^8S_{7/2} \rightarrow {}^6P_J$, ^6I_J, and ^6D_J. Co-doping of the $Li_2B_4O_7$:Gd and $LiKB_4O_7$:Gd glasses by Ag leads to significant (~ 100 times) increasing of intensity of the Gd^{3+} emission band at 311 nm. Such increasing of intensity of the emission band in the Gd-Ag co-doped borate glasses is explained by energy transfer from Ag^+ to Gd^{3+} centres under excitation with 273 nm that is resonant for the $4d^{10} \rightarrow 4d^9 5s^1$ ($^1S_0 \rightarrow {}^1D_2$) and $^8S_{7/2} \rightarrow {}^6I_J$ ($^6P_{7/2}$ level) transitions of the Ag+ and Gd^{3+} ions, respectively. Luminescence kinetics of the Gd^{3+} emission band at 311 nm in the $Li_2B_4O_7$:Gd,Ag and $LiKB_4O_7$:Gd,Ag (Gd_2O_3 1.0 mol. %, $AgNO_3$ 2.0 mol. %) glasses are satisfactory described by single exponential decay with lifetimes ~ 4.1 ms and ~ 4.2 ms, respectively. Thermal annealing of the Gd-Ag co-doped borate glasses in the air at $t_a = 460°C$ leads to additional increasing of intensity of the emission band at 311 nm. This effect can be explained by influence of Ag nanoparticles, which were formed during thermal annealing.

The obtained results clearly show that the investigated borate glasses co-doped with Gd and Ag are very promising materials for effective sources of UV radiation including solid-state UV lasers with working wavelength 311 nm ($^6P_{7/2} \rightarrow {}^8S_{7/2}$ channel).

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Self-energy of the Bose polaron

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Bose polaron is a quasiparticle which is formed as a result of the boson-impurity interaction between the impurity atom and the dilute Bose gas [4]. The information about characteristics of the Bose polaron (the damping of the impurity spectrum, the impurity effective mass, the immersion energy, etc.) is determined of the self-energy of the impurity. Although the calculation of such energy is a non-trivial problem, we obtained the diagrammatic representation of the self-energy in the second-order of the perturbation theory.

We tested the results of calculations of the self-energy on the example of the moving isotopes of helium in the Bose liquid [2]. In particular, our second-order prediction for the effective mass of ^{3}He equals 2.13 and is in agreement with the experimental value 2.15 [3], results of diffusion 2.20 [4] and variational 2.06 [5] Monte Carlo simulations.

Our analytical expression for the self-energy of the impurity can be used to describe the behavior of a single atom in the ultra-cold gases of alkali metals. Our method, which takes into account a certain class of Feynman's diagrams, can be extended onto the low-dimensional systems.

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Impurity states in the one-dimensional Bose gas

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The renewal of an interest (see for review Ref. [1]) to the problem of a single impurity atom (Bose polaron) immersed in the Bose-Einstein condensate is associated with growing opportunities of experimental techniques [2, 3]. Particularly in Refs. [4, 5] the possibility to control a small amount of impurity particles strongly coupled to the three-dimensional bosonic bath is demonstrated. The properties of the low-dimensional systems are less studied and only recently the results of quantum Monte Carlo simulations [6] and non-perturbative renormalization group approach [7] for the one-dimensional Bose polaron appeared.

In this talk the detailed discussion of the low-energy spectrum for a mobile impurity in the one-dimensional bosonic environment is given. We analyze only two analytically accessible limits, namely, the case of an impurity immersed in a dilute Bose gas, where one can use many-body perturbative techniques for low-dimensional bosonic systems, and the case of the Tonks-Girardeau (TG) gas, for which the usual fermionic diagrammatic expansion is applied.

Considering our system as a Fermi-Bose mixture with the vanishingly small fermionic density and by applying the perturbation theory up to the second order we have calculated binding energy and effective mass of the Bose polaron and found that the interaction with bosonic medium crucially changes the single-particle impurity Green's function providing the latter to exhibit a branch-point singularity. Using our second-order perturbative results we have proposed the general formula for the non-universal exponent determining this behavior. It is also demonstrated that the induced interaction, especially in the case of a large mass imbalance, has a profound effect on the behavior of a single impurity in the one-dimensional Bose gas.

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Telling apart *Felidae* and *Ursidae* from the distribution of nucleotides in mitochondrial DNA

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Rank–frequency distributions of nucleotide sequences in mitochondrial DNA [1] are defined in a way analogous to the linguistic approach, with the highest-frequent nucleobase serving as a whitespace [2]. For such sequences, entropy S and mean length $\langle L\rangle$ are calculated. These parameters are shown to discriminate the species of the *Felidae* (cats) and *Ursidae* (bears) families, see Fig. 1. From purely numerical values we are able to see in particular that giant pandas are bears while koalas are not. The observed linear relation between the parameters is explained using a simple probabilistic model. The approach based on the nonadditive generalization of the Bose-distribution [3] is used to analyze the frequency spectra of the nucleotide sequences. In this case, the separation of families is not very sharp. Nevertheless, the distributions for *Felidae* have on average longer tails comparing to *Ursidae*.

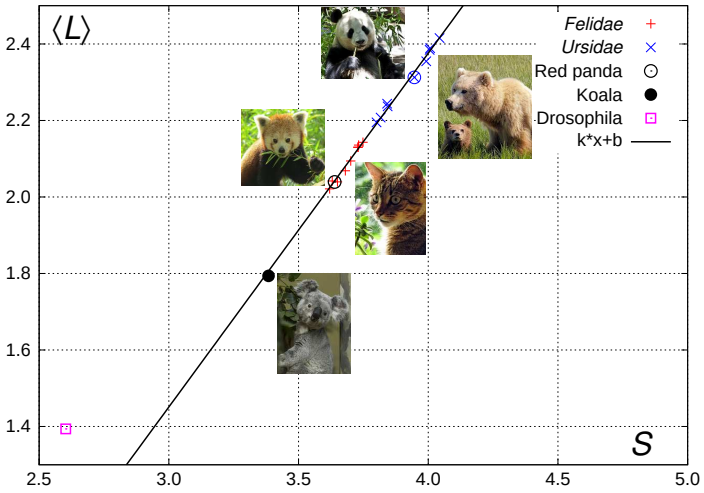

Figure 1. Entropy S and mean length $\langle L\rangle$ for families and species analyzed in the present work.

The proposed approach can be used in studies of mitochondrial genomes as the suggested set of parameters serve to discriminate animal families. The overall interest in this problem is caused by its interdisciplinary nature bridging several scientific domains, namely, biology, linguistics, and physics [4, 5].

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New exact analytic solutions to the extended KdV equation

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In this contribution we present analytic solutions to extended KdV equation (KdV2 in short) of the form

$$\eta_t + \eta_x + \frac{3}{2}\alpha\,\eta\eta_x + \frac{1}{6}\beta\,\eta_{3x} - \frac{3}{8}\alpha^2\eta^2\eta_x + \alpha\beta\left(\frac{23}{24}\eta_x\eta_{2x} + \frac{5}{12}\eta\eta_{3x}\right) + \frac{19}{360}\beta^2\eta_{5x} = 0. \tag{4}$$

This equation is obtained in second order approximation from the set of Euler's hydrodynamical equations for shallow water problem. Therefore the range of applicability of KdV2 is much wider than the applicability of KdV, which is first order in small parameters α and β (KdV can be obtained from (4) by neglection of second order terms).

Despite of nonintegrability of KdV2 we found analytic solutions to (1) in the following forms:

1. Single soliton solution [1]
$$\eta(x,t) = A\,\mathsf{Sech}^2[B(x-vt)] \tag{5}$$

2. Periodic (cnoidal) solution [2]
$$\eta(x,t) = A\,\mathsf{cn}^2[B(x-vt),m] + D \tag{6}$$

3. Periodic superposition solutions [3]
$$\eta_{\pm}(x,t) = \frac{A}{2}\left[\,\mathsf{dn}^2(B(x-vt),m) \pm \sqrt{m}\,\mathsf{cn}(B(x-vt),m)\,\mathsf{dn}(B(x-vt),m)\right] + D. \tag{7}$$

In all these cases the coefficients of solutions A, B, D, v are given explicitly as functions of parameters α, β and m. Physical limitations imposed by periodicity and volume (mass) conservation are discussed.

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Two cases of GPS pulsars: J1740+1000 and B1800−21

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Most of the pulsar radio spectra can be described by a simple power-law function. However, spectra of some pulsars show a different behaviour, among them are the so-called gigahertz-peaked spectra (GPS) characterized by a positive spectral index in the frequency range below one gigahertz (Kijak et al. 2017). The analysis of the spectral evolution of a binary pulsar PSR B1259-63 provided evidence that GPS have rather some environmental origin than an intrinsic one (Kijak et al 2011). Recently the model of the free-free thermal absorption was used in order to explain the turnover around one gigahertz in pulsars spectra by Lewandowski et al. (2015) and Rajwade et al. (2016). Such absorption may be caused by pulsars environments such as pulsar wind nebulae, dense filaments in supernovae remnants or cooler H II regions.
In my talk I will focus on two unusual pulsars that show a turn-over around 1 GHz: J1740+1000 and B1800-21. In both cases the previous measurements have brought results that yield to intractable discrepancy between different interpretations of the nature of their spectra. For that reason both pulsars were observed at frequency range from 325 MHz to 5 GHz. I will present the results of our recent observations and discuss the shape of the newly acquired spectra. The first part of our observational project involved observations of the two pulsars J1740+1000 and B1800-21 using the Giant Meterwave Radio Telescope (GMRT) located near Pune in India. GMRT consist of 30 parabolic antennas spread in a Y shape over distances up to 25 km. Each of the antennas has 45 m dish diameter and is fully steerable. In the second part observations were made using the Green Bank Telescope that is a fully steerable single dish with 100-meter diameter collecting area and is located in Green Bank (West Virginia, USA).
To summarize I will present the result of the recent multiwavelength radio observations of two pulsars J1740+1000 and B1800-21. In case of J1740+1000 the recent data show that this is a GPS pulsar. In case of B1800-21 we confirm its spectrum shape as GPS pulsar.

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Symmetries and conservation laws for the Karczewska–Rozmej–Rutkowski–Infeld equation

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We give a complete description of generalized symmetries and local conservation laws for the fifth-order Karczewska–Rozmej–Rutkowski–Infeld equation for shallow water waves in a channel with variable depth.

In particular, we show that this equation has no genuinely generalized symmetries and thus is not symmetry integrable.

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Diode laser optical pumping of a tunable single-mode dye laser

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A tunable continuous wave single-mode dye laser, operated in green spectral region, directly optically pumped by a low-cost diode laser at $\lambda = 445$ nm, is reported [1].

Laser radiation in green spectral region is not easily achieved, in particular in single-mode regime. In our laser spectroscopic investigations of the hyperfine structure and isotope shifts in selected rare earths elements the possibility of excitation of the electronic transitions in the green region was much desired for verification of certain data based on investigations in more easily accessible spectral regions; this was the main motivation for setting up of this laser. We were encouraged by the proposal and first successful attempts of the diode laser pumping of dye lasers, reported in the 90-ties [2-5], and by the recent availability of high-power blue diode lasers.

Another research group is currently working on a pulsed dye laser system, optically pumped with $\lambda = 445$ nm diode lasers [e.g. 6].

In the development of our laser we applied a modified Coherent CR 699-21 ring laser system. Originally the laser was operated on the dye Coumarin 498; the wavelength tuning range ca. 485-530 nm was achieved with power levels up to ca. 350 mW in broad-band operation and up to ca. 250 mW in single-mode regime, when pumped with nearly 4 W of diode laser radiation. The single-mode operation proved fairly stable and the frequency could be smoothly scanned over ca. 40 GHz. The laser was applied in spectroscopic investigations of the hyperfine structure in terbium [e.g. 7] and holmium.

Currently attempts to extend the generation range of the laser towards longer wavelengths with the use of other coumarin dyes are in progress.

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Thermodynamics of a slowly rotating Einstein-Maxwell-dilaton black hole

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We have found a slowly rotating black hole solution in multidimensional Einstein-Maxwell-dilaton gravity with dilaton potential of Liouville-type. The obtained solution exists for different type of dilaton coupling. Temperature and entropy for this solution has been obtained and corresponding black hole thermodynamics has been investigated. Thermodynamical behaviour can be considered in an extended space when one treats the cosmological constant as a thermodynamical pressure, this approach allows to obtain corresponding equation of state which is similar to well known Van der Waals equation [1]. Using this approach we have obtained the equation of state for our black hole solution and investigated its critical behaviour.

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Spectroscopic properties of chromium-doped $LaAlO_3$ nanocrystals

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$LaAlO_3$ nanocrystals doped with Cr^{3+} (1.0 wt%) ions was prepared by Pechini method. Sample has been investigated using methods of spectroscopy including optical absorption, photoluminescence and electron paramagnetic resonance. Absorption, emission, and luminescence kinetics of the observed emission were measured low- (77K) and room-temperature (300K) in the IR, visible and near UV range. Energy levels reffering to experimentally observed transitions have been located based on absorption measurements. Recorded Cr^{3+} emission spectrum presents typical strong crystal field transition. The emission decay profile presents multi-exponential character.

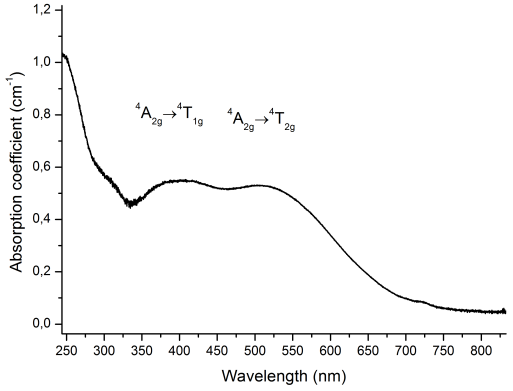

Figure 2. Absorption spectrum of $LaAlO_3$ nanocrystals doped with chromium ions measured room temperature.

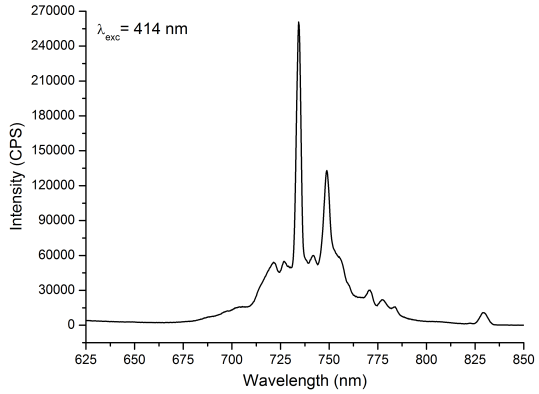

Figure 3. Emission spectrum of $LaAlO_3$ nanocrystals doped with chromium ions measured room temperature.

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Integrability analysis of the chaotic and hyperchaotic financial models

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We consider two chaotic finance models recently studied in a literature. The first one, introduced by the Huang and Li [1], has a form of a three first order nonlinear differential equations

$$\dot{x} = z + (y - a)x, \quad \dot{y} = 1 - by - x^2, \quad \dot{z} = -x - cz.$$

The second system, called a hyperchaotic finance model [2], is defined by

$$\dot{x} = z + (y - a)x + u, \quad \dot{y} = 1 - by - x^2, \quad \dot{z} = -x - cz, \quad \dot{u} = -dxy - ku.$$

In the both models (a, b, c, d, k) are real positive parameters. We show that the Huang–Li system is not integrable in a class of functions meromorphic in variables (x, y, z) for all real values of parameters (a, b, c) while the hyperchaotic system is not integrable in the case when $k = c$ and $\Delta := 1 + d(a + d - c) > 0$. We give analytic proofs of this facts analyzing properties the of differential Galois groups of variational equations along certain particular solutions. On the other-hand, we show that for certain sets of the parameters (a, b, c, d, k), when $\Delta \leq 0$, the hyperchaotic system possesses polynomial first integral, which can be use to reduce the dimension of the system by one.

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Nonlinear mechanics of cosmological models

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The methods of dynamical systems are applied to study of dynamics of cosmological models with the Robertson-Walker symmetry and barotropic equation of state. We demonstrate that there is a large class of models which dynamics reduces to the form of dynamical systems of Newtonian type. The notion of structural stability is discussed in this context.

We study dynamical complexity of the Lemaître-Friedmann-Robertson-Walker cosmological models. As a source of gravity we consider both the minimally and non-minimally coupled to gravity scalar field as well as the barotropic matter. In study of evolutional paths of cosmological models the methods of dynamical systems is used.

The issues of structural stability of the systems is investigated in the context of study of generic and non-generic models in the ensemble of dark energy FRW models. We distinguish typical and non-typical evolutional scenarios and argue that a good model of the Universe cannot be necessary generic, i.e. the universe model can be fine-tuned like on the knife edge.

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Curvature and torsion of quantum evolution

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We study geometric characteristics of quantum evolution, namely curvature and torsion [1]. The curvature shows a deviation of the state vector in quantum evolution from the geodesic line. The torsion shows a deviation of state vector from the plane of evolution (a two-dimensional subspace) at a given time. These characteristics of quantum evolution are useful for studies of brachistochrone problem, entanglement and others. Some examples are presented.

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Quantum face of decay processes: Unstable systems in rest and in motion

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From fundamental principles it follows that the modulus of the survival amplitude, $|a(t)|$, can not have an exponential form: Khalfin in 1957 assuming that the spectrum of the Hamiltonian of the system considered is bounded from below and using the integral representation of $a(t)$ as the Fourier transform of the energy distribution function, $\omega(E)$ proved that $|a(t)| \to 0$ as $t \to \infty$ slower that any exponential function of time t [1]. Nevertheless results of numerical model calculations presented in the literature show that the survival probability $\mathcal{P}(t) = |a(t)|^2$ has the exponential form starting from times much smaller than the lifetime τ up to times $t \gg \tau$, and that $\mathcal{P}(t)$ exhibits inverse power–law behavior at the late time region for times longer

than the so–called crossover time $T \gg \tau$ (The crossover time T is the time when the late time deviations of $\mathcal{P}(t)$ from the exponential form begin to dominate). This effect was confirmed experimentally not long ago [2]. This is why physicists believe that the survival probability has the exponential form for times much smaller than T and that nonexponential deviations manifests itself only for suitable long times. More detailed analysis of the problem shows that in fact the survival probability $\mathcal{P}(t)$ can not take that exponential form at any time interval including times smaller than the lifetime τ: The exponential form of $\mathcal{P}(t)$ for $t < \tau$ and $t \sim \tau$ obtained numerically for models of quantum unstable systems is only a better or worse approximation. In fact for times $t \sim \tau$ and for the later times the form of $\mathcal{P}(t)$ looks as a composition of an oscillating function and exponential function, that is it has an oscillating form. The amplitude of these oscillations is very small for $t \ll \tau$ and grows with increasing time and depends on the model considered, that is on the properties of the energy distribution function, $\omega(E)$. One of the consequences of this effect is that the instantaneous energy (mass) of the unstable system can not be constant in time.

We also study the survival probability of moving relativistic unstable particles. We show that assumption that velocity of such particles is constant leads to the wrong result contrary to claims of authors of [3]. Following [4] we assume that particles move with definite momentum $\vec{p} \neq 0$ and we found decay curves of such particles for the quantum mechanical models considered. These model studies show that late time deviations of the survival probability of these particles from the exponential–like form of the decay law, that is the transition times region between exponential–like and non-exponential form of the survival probability, should occur much earlier than it follows from the classical standard approach resolving itself into replacing time t by t/γ (where γ is the relativistic Lorentz factor) in the formula for the survival probability and that the survival probabilities should tend to zero as $t \to \infty$ much slower than one would expect using classical time dilation relation. We show that analogously to the particles decaying in the rest system fluctuations of $\mathcal{P}(t)$ take also place in the case of moving particles, but the amplitude of these fluctuations is larger than that in the the case of decays in the rest system.

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Quantum supersymmetry harmonic oscillator in a Penning trap

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The Pauli equation describing a spin-$1/2$ charged particle in a processing chamber of an ideal Penning trap is considered. To solve the radial wave equation we apply the Witten's model of supersymmetric quantum mechanics [1]. We introduce the specific ladder operators and exploit the relations between two shape invariant partner potentials and phase shift of the partner Hamiltonians associated with them [2]. We establish the SUSY QM superalgebra.

The first order relativistic approximation of the Dirac equation for a spin-1/2 charge in a Penning trap is analyzed. The first order perturbation theory as expansion in powers of c^{-2} is applied. Having used the Birkhoff transformation [3] we present relativistic perturbations as functions of action-angle variables. Standard quantization by means of creation and annihilation operators gives accurate corrections to energy levels of the charge in an ideal Penning trap. Resonances which produce degenerate states are considered. The relativistic corrections to their energy spectra are found with the help of the Birkhoff transformation.

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